

Reg. No. :

Name :

**Fourth Semester B.Tech. Degree Examination, June 2016
(2013 Scheme)**

13.401 : ENGINEERING MATHEMATICS – III (E)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Define analyticity of a function at a point. Show that $f(z) = z\bar{z} - 2i$ is not analytic at $z = 0$.

2. Use Cauchy's integral formula to evaluate $\oint_C \frac{z+2}{2z^2-z} dz$ where C is the circle

$$|z| = \frac{1}{4}.$$

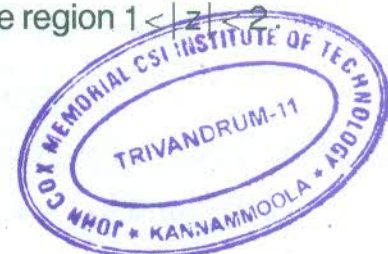
3. Expand the function $f(z) = \frac{1}{z(z-2)}$ as a series about $z = 0$ in the region $1 < |z| < 2$.

4. Explain the terms :

i) objective function ii) feasible solution

iii) basic feasible solution iv) optimal solution of an LPP.

5. Find the angle between the vectors $u = (1, 3, -5)$ and $v = (2, -3, 4)$ in \mathbb{R}^3 .



PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – 1

6. a) Test the differentiability of the function $f(z)$ at $z = 0$ if

$$f(z) = \begin{cases} \frac{xy^2}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

P.T.O.



- b) Test if $v = e^x (x \sin y + y \cos y)$ is harmonic. If so, find the analytic function $f(z) = u + iv$.
- c) Sketch the image of the region $-\frac{\pi}{4} < y < \frac{\pi}{4}$ under the map $w = e^z$.
7. a) Show that the function $f(z) = \sin 3z$ is analytic everywhere.
- b) If $u = \sin x \cosh y$, show that u is a harmonic function.
- c) Find a bilinear transformation which maps the points $-1, 0, 1$ onto $0, 1, -1$ respectively. Find the image of $z = i$ under this transformation.

Module - 2

8. a) Evaluate $\int_0^3 (x^2 + \bar{z}) dz$ along Y axis from 0 to $2i$ and then from $2i$ to 3 along the line $2x + 3y = 6$.
- b) Evaluate $\oint_C \frac{ze^z}{(2z-1)^2(3z-2)} dz$ where C is the unit circle $|z| = 1$.
- c) Expand $f(z) = \sin z$ in Taylor's series about $z = \frac{\pi}{4}$.
9. a) Expand $f(z) = \frac{z-1}{z^2+2z}$ as a Taylor series about $z = -1$.
- b) Find the residue of $f(z) = \frac{z^2}{(2z-1)^2(z+3)^2}$ at $z = \frac{1}{2}$.
- c) Evaluate $\int_0^\pi \frac{d\theta}{5 + \cos \theta}$ using contour integration.



Module - 3

10. a) Solve the LPP :

$$\text{Maximize } z = 5x_1 + 8x_2$$

Subject to

$$3x_1 + 2x_2 \geq 3$$

$$x_1 + 4x_2 \geq 4$$

$$x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

b) Use Big-M method to solve the LPP :

$$\text{Minimize } z = 9x_1 + 10x_2$$

Subject to

$$x_1 + 2x_2 \geq 25$$

$$4x_1 + 3x_2 \geq 24$$

$$3x_1 + 2x_2 \geq 60$$

$$x_1, x_2 \geq 0$$



11. a) Solve the LPP :

$$\text{Maximize } z = 2x_1 + x_2$$

Subject to

$$4x_1 + 3x_2 \leq 12$$

$$4x_1 + x_2 \leq 8$$

$$4x_1 - x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

b) Solve the LPP, by Big-M method

$$\text{Minimize } z = 4x_1 + x_2$$

Subject to

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

**Module - 4**

12. a) Show that the span of three vectors v_1, v_2, v_3 in a vector space V is a subspace of V .
- b) Write $w_1 = (-1, -3, 0)$ as a linear combination of $w_2 = (1, -1, 2)$ and $w_3 = (2, 0, 3)$. Can $\{w_1, w_2, w_3\}$ be a basis for \mathbb{R}^3 ? Justify your answer.
- c) Show that the vectors $(1, -1, 2), (2, 0, 3), (-4, -2, 3), (-1, -3, 0)$ spans \mathbb{R}^3 ; but it is not a basis for \mathbb{R}^3 .
13. a) Define the null space of a linear transformation. Show that the null space of $T: V \rightarrow W$, where V and W are vector spaces, is a subspace of V .
- b) Find the null space and nullity of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, 0, x + y)$.
- c) Find an orthonormal basis for \mathbb{R}^3 from $(1, 0, 1), (1, 0, 0)$ and $(2, 1, 0)$.
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