Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, June 2016
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (E)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Define analyticity of a function at a point. Show that $f(z) = z\bar{z} - 2i$ is not analytic at $z = 0$.

2. Use Cauchy's integral formula to evaluate $\oint_C \frac{z+2}{2z^2 - z}\, dz$ where C is the circle $|z| = \frac{1}{4}$.

3. Expand the function $f(z) = \frac{1}{z(z-2)}$ as a series about $z = 0$ in the region $1 < |z| < 2$.

4. Explain the terms :

   i) objective function ii) feasible solution

   iii) basic feasible solution iv) optimal solution of an LPP.

5. Find the angle between the vectors $u = (1, 3, -5)$ and $v = (2, -3, 4)$ in $\mathbb{R}^3$.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – 1

6. a) Test the differentiability of the function f(z) at z = 0 if

$$f(z) = \begin{cases} \frac{xy^2}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

P.T.O.

b) Test if $v = e^x (x \sin y + y \cos y)$ is harmonic. If so, find the analytic function $f(z) = u + iv$.

c) Sketch the image of the region $-\frac{\pi}{4} < y < \frac{\pi}{4}$ under the map $w = e^z$.

7. a) Show that the function $f(z) = \sin 3z$ is analytic everywhere.

b) If $u = \sin x \cosh y$, show that u is a harmonic function.

c) Find a bilinear transformation which maps the points –1, 0, 1 onto 0, 1, –1 respectively. Find the image of $z = i$ under this transformation.

Module – 2

8. a) Evaluate $\int_0^3 (x^2 + \bar{z})dz$ along Y axis from 0 to 2i and then from 2i to 3 along the line $2x + 3y = 6$.

b) Evaluate $\oint_C \frac{ze^z}{(2z-1)^2(3z-2)} dz$ where C is the unit circle $|z| = 1$.

c) Expand $f(z) = \sin z$ in Taylor's series about $z = \pi/4$.

9. a) Expand $f(z) = \frac{z-1}{z^2 + 2z}$ as a Taylor series about $z = -1$.

b) Find the residue of $f(z) = \frac{z^2}{(2z-1)^2(z+3)^2}$ at $z = \frac{1}{2}$.

c) Evaluate $\int_0^\pi \frac{d\theta}{5 + \cos\theta}$ using contour integration.

Module – 3

10. a) Solve the LPP :

Maximize $z = 5x_1 + 8x_2$

Subject to

$$3x_1 + 2x_2 \geq 3$$
$$x_1 + 4x_2 \geq 4$$
$$x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

b) Use Big-M method to solve the LPP :

Minimize $z = 9x_1 + 10x_2$

Subject to

$$x_1 + 2x_2 \geq 25$$
$$4x_1 + 3x_2 \geq 24$$
$$3x_1 + 2x_2 \geq 60$$
$$x_1, x_2 \geq 0$$

11. a) Solve the LPP :

Maximize $z = 2x_1 + x_2$

Subject to

$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + x_2 \leq 8$$
$$4x_1 - x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

b) Solve the LPP, by Big-M method

Minimize $z = 4x_1 + x_2$

Subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Module – 4

12. a) Show that the span of three vectors v_1, v_2, v_3 in a vector space V is a subspace of V.

    b) Write $w_1 = (-1, -3, 0)$ as a linear combination of $w_2 = (1, -1, 2)$ and $w_3 = (2, 0, 3)$. Can $\{w_1, w_2, w_3\}$ be a basis for $\mathbb{R}^3$? Justify your answer.

    c) Show that the vectors (1, –1, 2), (2, 0, 3), (–4, –2, 3), (–1, –3, 0) spans $\mathbb{R}^3$; but it is not a basis for $\mathbb{R}^3$.

13. a) Define the null space of a linear transformation. Show that the null space of $T : V \to W$, where V and W are vector spaces, is a subspace of V.

    b) Find the null space and nullity of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, 0, x + y)$.

    c) Find an orthonormal basis for $\mathbb{R}^3$ from (1, 0, 1), (1, 0, 0) and (2, 1, 0).
